Signal Energy Measures and Metrics

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Description

I propose several measures and metrics based on the cummulative energy content of time signals. Similarity, in this case, is not based directly on geometric comparisons but on signal energy transfer.

Mathematical Principles

Consider the sequence $A = \{a_1, a_2, ..., a_n\}$ of real-valued signal amplitudes and the corresponding sequence of time stamps $T = \{t_1, t_2, ..., t_n\}$, where $t_i < t_{i+1}$. For this discussion I will use the unitless time convention $t_i = i$. Two example sequences are shown in Figure 1. For these two examples, $n = 4500$ and the signal onset time is at t_{165}. Zero mean Gaussian noise of variance 2 corrupts an otherwise clean signal of Fourier components of frequencies $\omega_g = 2\pi(3g-1), g = 1, ..., 10$ and an envelope function $(t - t_{165}) exp(-(t - t_{165})/120)$. The difference in the two signals is a small perturbation on the Fourier amplitude coefficients.

I define the unnormalized cummulative signal energy signal as $E_{i,j} = \{e_k\}, k = 1, ..., j - i + 1$ where $e_k = \sum_{m=i}^{k} a_m^2$. We also define a relative energy signal $\bar{E}_{i,j} = \{e_k\}/e_j, k = 1, ..., j - i + 1$, and a relative time signal $\bar{T}_{i,j} = \{k\}/(j - i + 1), k = 1, ..., j - i + 1$. And finally, we introduce the energy envelope signal $\bar{F}_{i,j} = (1 - \bar{E}_{i,j})^{1/2}$ of elements $\{f_k\}$. Note that the energy envelope signature $\bar{F}_{i,j}$ is proportional to the true signal envelope of A in the limit of an exponentially decaying cosine of infinite frequency.

Examples of $\bar{F}$ are shown in Figure 2 for the two example sequences using two different time windows. Each sequence is normalized to an initial value of 100, units of percent. At upper left are the the energy envelopes corresponding to a time window of 200 ticks beginning at t_0. At lower left are the envelopes corresponding to a time window of 700 ticks beginning at t_0. Examples of the ratio $\bar{F}/\bar{T}$ are also shown in Figure 2. At upper (lower) right are these ratio sequences corresponding to a time window of 200 (700) ticks from t_0. Then general similarity of the two example sequences is apparent from both figures.

I propose three initial simple measures on a single data sequence given a known time of arrival $t_0 = i$.

- $m^{(1)}_{i,j} = E_{i,j}(k)$ is the total signal energy in the interval $i \le t \le j$.
- $m^{(2)}_{i,j} = max(\bar{F}_{i,j}(k)/\bar{T}_{i,j}(k))$ is the maximum elementwise ratio of the envelope signal to the time signal.
- $m^{(3)}_{i,j} = k + i - 1$ (where k satisfies $m^{(2)}_{i,j}$) is the time of occurence of $m^{(2)}_{i,j}$.

I propose a single metric for comparing two energy envelope functions

- $M^{(1)}_{i,j} = \sum_{k=1}^{j-i+1} |f^1_k - f^2_k| f^2_k$ is the weighted sum of differences between signal amplitudes.

There are many possible ways of constructing measures and metrics on the sequences $\bar{F}$, $\bar{E}$, and $\bar{T}$. I have presented four that I believe may capture much about the character of any sequence, especially if considered at several time scales. It was my initial intention to use a DTW (dynamic time warping) metric to measure the difference between energy envelopes. The results for monotonic interger-valued signatures were perplexing. I expect that DTW distances will be added in the future when I can get meaningful results. The numerical results for the two examples curves are given in the following table.

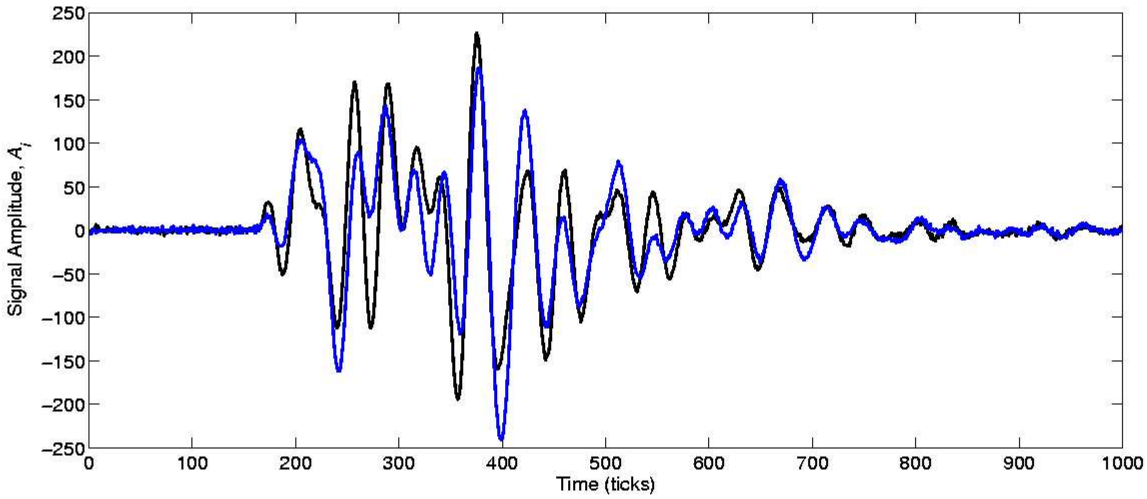

Figure 1: Example synthetic time traces with Gaussian noise. The time of signal onset is $t_0 = 165$.

	blue		black
t_0	165		166
$m^{(1)}_{i,i+199}$	1.41		1.01
$m^{(1)}_{i,i+699}$	2.82		2.69
$m^{(2)}_{i,i+199}$	1.01		1.22
$m^{(2)}_{i,i+699}$	2.28		2.23
$m^{(3)}_{i,i+199}$	198		130
$m^{(3)}_{i,i+699}$	239		263
$M^{(1)}_{i,i+199}$		11.5	
$M^{(1)}_{i,i+699}$		1.82	

Physical and Engineering Principles

The measures and metrics presented here are an attempt to capture and quantify the time evolution of signal energy. Abstractly, this idea can be applied to any signal as measures of similarity. In some cases, it may be easy to identify signal energy with familiar physical quantities. For example, accelerometer data can be related to kinetic energy. Some notable signal differences that do not significantly affect the energy signal are polarity reversals, low amplitude frequency components, and time of signal arrival. In some cases, according to data analysts, metrics should be insensitive to these features.

Usage

The Matlab script (Emet) that runs the algorithm is reproduced below. There is a single user-defined parameter that indicates whether or not to subtract signal means. This option should be used cautiously with short signals for which might reasonably be expected to have a large bias. It is often best for the user to presubtract a global mean before calling Emet.

It is assumed that the input signals are truncated beginning at a given or computed time of arrival and ending at a time defined by the user. If several time scales are of interest, then Emet must be called once for each time scale.

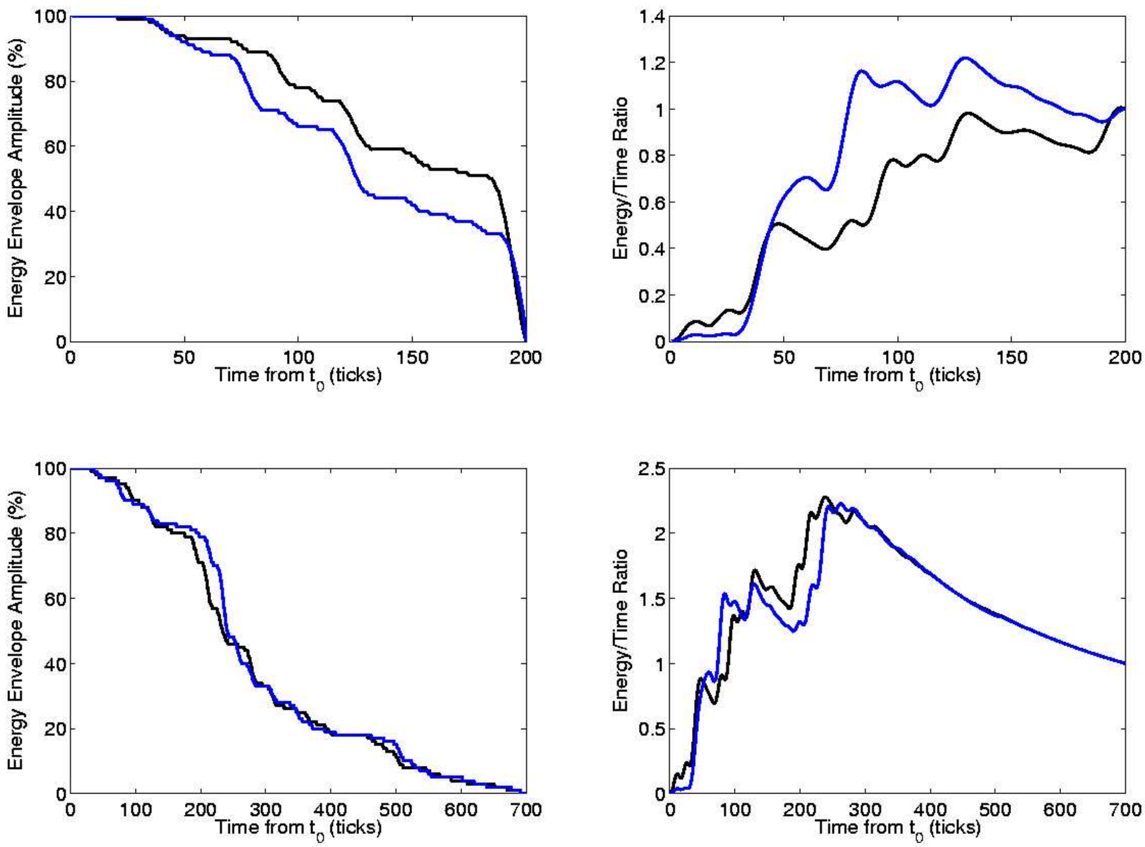

Figure 2: Example energy envelope sequences. For each of the two example sequences: $\bar{F}_{i,i+199}$ (upper left), $\bar{F}_{i,i+699}$ (lower left), $\bar{F}_{i,i+199}/\bar{T}_{i,i+199}$ (upper right), $\bar{F}_{i,i+699}/\bar{T}_{i,i+699}$ (lower right).

```
function [dist,Etot,ETmax,ETtime,Q]= Emet(ref,data,mflg)

%
% EMET compares two input signals for engery content
% and returns one metric comparison value: the DTW
% (dynamic time warping) distance and several energy
% based mesures on the signal inputs.
% Tom Asaki (667-5787) (asaki@lanl.gov)
%
% USAGE:
% [dist,Etot,ETmax,ETtime,Q]= Emet(ref,data)
% [dist,Etot,ETmax,ETtime,Q]= Emet(ref,data,mflg)
%
% INPUTS:
% ref   a vector data signal used as a reference
% data  a vector data signal for evaluation relative
%       to the reference.
% mflg  a flag for indicating whether or not to
%       subtract the signal means.  Default = 0.
%
% OUTPUTS:
% dist  the DTW distance between the energy content
%       signatures of data and ref.  Comparison is
%       performed on the normalized profiles.
%       *** dist is currently not DTW, it is instead
%       *** a weighted integral difference in profiles
% Etot  the total energy content of input signals.
%       It is a vector of two values:
%       [energy_ref energy_data]
% ETmax the the maximum ratio of the relative energy
%       to the relative time in each signal.  ETmax
%       is a vector of two values:
%       [ratio_ref ratio_data]
% ETtime is the time of occurence of ETmax.  ETtime
%       is a vector of two values:
%       [time_ref time_data]
% Q     is a cell array that returns extra diagnostic
%       information for the user.
%
% NOTES:
% (1) data and ref need not have equal length for the
% routine to function.  But the DTW algortithm will
% penalize this length difference, and other comparisons
% may be problematic.  Have some caution.
%

%%%%% INITIALIZATIONS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% set defaults
if nargin<3; mflg=0; end

% columize inputs
ref=ref(:);
data=data(:);

%%%%% MAIN ROUTINE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% substract mean signal value
if mflg==1
    ref=ref-mean(ref);
    data=data-mean(data);
end

% compute energy profile of reference
% and the energy 'envelope'
ERef=cumsum(ref.^2);
ERefProfile=round(100*sqrt(1-ERef/ERef(end)));

% compute energy profile of data
% and the energy 'envelope'
EDat=cumsum(data.^2);
EDatProfile=round(100*sqrt(1-EDat/EDat(end)));

% compute the DTW distance between envelopes
%[Dist,Dist2,D,k,w]=dtwfast(EDatProfile,ERefProfile);
%dist=Dist2;
dist=sum(abs(EDatProfile-ERefProfile).*(ERefProfile/50))/length(ref);

% find total energies
Etot(1)=ERef(end);
Etot(2)=EDat(end);

% find ETmax and ETtime results
ETref=(ERef/Etot(1))./((1:length(ref))'/length(ref));
[temp,idx]=sort(ETref);
ETmax(1)=temp(end);
ETtime(1)=idx(end);
ETdat=(EDat/Etot(2))./((1:length(data))'/length(data));
[temp,idx]=sort(ETdat);
ETmax(2)=temp(end);
ETtime(2)=idx(end);

Q{1}=ERefProfile;
Q{2}=EDatProfile;
Q{3}=ETref;
Q{4}=ETdat;

return
```